Scheduling Algorithm and Analysis

Rate Monotonic
(Module 29)

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Rate-Monotonic Scheduling Algorithm

- A base case: no additional overhead, simple periodic tasks with $p_i = D_i$
- Assign priorities according their periods
  - $T_i$ has a higher priority than $T_k$ if $i < k$ ($p_i < p_k$)
  - Is RM optimal? ⇒ if there is a feasible fixed-priority schedule, then RM is feasible
  - How do we know RM is feasible ⇒ schedulability test

- Results:
  - RM is optimal if $p_i \geq D_i$
  - sufficient condition ⇒ utilization test

\[ U = \sum_{i=1}^{n} \frac{e_i}{p_i} \leq n(2^{1/n} - 1) \]

- a complete test ⇒ what is the worst response time given all possible arrivals and preemptions
Critical Instant

- Critical instant of $T_i$: a job of $T_i$ arriving at the instant has a maximum response time
- If we can find the critical instant of $T_i$, then
  - check whether all jobs of $T_i$ meet their deadlines
  - let’s increase $e_i$ until the maximum response time = $D_i$
  - \( \Rightarrow \) schedulable utilization
- In-phase instant is critical: all higher priority tasks are released at the same instant (assume all jobs are completed before the next job in the task is released.)
  - which $T_2$ has the maximum response time
Schedulability: UB Test

- Utilization bound (UB) test: a set of $n$ independent periodic tasks scheduled by the rate monotonic algorithm will always meet its deadlines, for all task phasings, if

$$\frac{C_1}{T_1} + \ldots + \frac{C_n}{T_n} \leq U(n) = n(2^{1/n} - 1)$$

- For harmonic task sets, the utilization bound is $U(n)=1.00$ for all $n$.

<table>
<thead>
<tr>
<th>$U(1)$</th>
<th>$U(2)$</th>
<th>$U(3)$</th>
<th>$U(4)$</th>
<th>$U(5)$</th>
<th>$U(6)$</th>
<th>$U(7)$</th>
<th>$U(8)$</th>
<th>$U(9)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.828</td>
<td>0.779</td>
<td>0.756</td>
<td>0.743</td>
<td>0.734</td>
<td>0.728</td>
<td>0.724</td>
<td>0.720</td>
</tr>
</tbody>
</table>
Schedulability Test: Time-Demand Analysis

- Consider in-phase instant only
- If \( J_i \) is done at \( t \), then the total work must be done in \([0, t]\) is (from \( J_i \) and all higher priority tasks)

\[
w_i(t) = e_i + \sum_{k=1}^{i-1} \left\lceil \frac{t}{p_k} \right\rceil e_k
\]

- Can we find a \( t \leq D_i \) such that \( w_i(t) \leq t \)
  - cannot check all \( t \in [0, D_i] \)
  - check all arrival instants and \( D_i \)
- The completion time of \( J_i \) satisfies

\[
t = e_i + \sum_{k=1}^{i-1} \left\lceil \frac{t}{p_k} \right\rceil e_k
\]
Supplementary Slides